

# QCD Vacuum Topology and Glueballs

Hilmar Forkel

*IFT - Universidade Estadual Paulista, Rua Pamplona, 145, 01405-900 Sao Paulo, SP, Brazil, and  
Institut für Theoretische Physik, Universität Heidelberg, D-69120 Heidelberg, Germany*

**Abstract.** We outline a comprehensive study of spin-0 glueball properties which, in particular, keeps track of the topological gluon structure. Specifically, we implement (semi-hard) topological instanton physics as well as topological charge screening in the QCD vacuum into the operator product expansion (OPE) of the glueball correlators. A realistic instanton size distribution and the (gauge-invariant) renormalization of the instanton contributions are also implemented. Predictions for  $0^{++}$  and  $0^{-+}$  glueball properties are presented.

## INTRODUCTION

While the naive quark-model description of the classical hadrons is surprisingly successful, we lack an equally simple and systematic approach to hadrons which are not predominantly built from valence quarks. Glueballs [1], containing no valence quarks at all, are a case in point: although “valence gluons” had a limited amount of success in glueball models, their role as relevant degrees of freedom (as opposed to strings, e.g.) is much less clear. This is partly because, in contrast to valence quarks, gluons cannot be traced by observable internal quantum numbers. In the following, we sketch a recent study of glueballs which instead relies on some robust topological gluon properties to keep track of the elusive hadronic glue. A comprehensive analysis of both spin-0 glueball channels along these lines can be found in Ref. [2].

## SKETCH OF APPROACH

Our approach is based on the correlations functions of the scalar ( $0^{++}$ ) and pseudoscalar ( $0^{-+}$ ) glueball channels,

$$\Pi_G(-q^2) = i \int d^4x e^{iqx} \langle 0 | T O_G(x) O_G(0) | 0 \rangle \quad (1)$$

where  $O_G$  with  $G \in \{S, P\}$  are the standard gluonic interpolating fields (with lowest mass dimension):  $O_S(x) = \alpha_s G_{\mu\nu}^a(x) G^{a\mu\nu}(x)$  and  $O_P(x) = \alpha_s G_{\mu\nu}^a(x) \tilde{G}^{a\mu\nu}(x)$ . The zero-momentum limits of these correlators are governed by low-energy theorems (LETs) [3] which impose stringent constraints on the sum rules and require nonperturbative short-distance physics [4, 2].

Our theoretical framework for calculating the glueball correlators at short distances, i.e. large, spacelike momenta  $Q^2 \equiv -q^2 \gg \Lambda_{QCD}$ , is the instanton-improved operator

product expansion (IOPE)

$$\Pi_G(Q^2) = \sum_{D=0,4,\dots} \tilde{C}_D^{(G)}(Q^2; \mu) \langle \hat{O}_D \rangle_\mu, \quad (2)$$

which factorizes “hard” mode (with momenta  $|k| > \mu$ ) and “soft” mode contributions (with  $|k| \leq \mu$ ). The perturbative Wilson coefficients are standard. In Ref. [2] we calculate and study nonperturbative contributions due to direct instantons [5] and topological-charge screening.

In order to make contact with the hadronic information contained in the glueball correlators, one writes the dispersive representation

$$\Pi_G(Q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{Im\Pi_G(-s)}{s+Q^2} \quad (3)$$

(necessary subtractions implied) and inserts the standard sum-rule description of the spectral functions,

$$Im\Pi_G^{(ph)}(s) = Im\Pi_G^{(pole)}(s) + Im\Pi_G^{(cont)}(s), \quad (4)$$

which contains one or two resonance poles  $Im\Pi^{(pole)}(s) = \pi \sum_{i=1}^2 f_{Gi}^2 m_{Gi}^4 \delta(s - m_{Gi}^2)$  and the local-duality continuum  $Im\Pi_G^{(cont)}(s) = \theta(s - s_0) Im\Pi_G^{(IOPE)}(s)$ .

One then writes QCD sum rules by matching the Borel-transformed IOPE correlators - weighted with powers of  $-Q^2$  - to these phenomenological counterparts, whose glueball parameters are thereby determined.

## RESULTS AND CONCLUSIONS

Our study [2] has produced several remarkable and phenomenologically important results. Contrary to naive expectation, much of the topological gluon physics was found to happen at surprisingly short distances  $|x| \sim 0.2 - 0.3$  fm. To study its impact on glueball properties, we have performed a comprehensive quantitative analysis of eight Borel sum rules. The results reveal a rather diverse pattern of spin-0 glueball properties.

In the scalar channel, the improved treatment of the direct-instanton sector reduces our earlier (spike-distribution based) result for the  $0^{++}$  glueball mass by about 20%, to  $m_S = 1.25 \pm 0.2$  GeV. This is somewhat smaller than the quenched lattice results [6] which are, however, expected to be reduced by light-quark effects and quarkonium admixtures. Our mass prediction is consistent with the broad glueball state found in a recent  $K$ -matrix analysis [7] (including the new Crystal Barrel states). The systematics in our results from different Borel-moment sum rules indicates a rather large width of the scalar glueball,  $\Gamma_S \sim 0.3$  GeV. Our prediction for the glueball decay constant,  $f_S = 1.05 \pm 0.1$  GeV, is several times larger than the value obtained when ignoring the nonperturbative Wilson coefficients. The implied, small glueball size is in agreement with lattice results. Furthermore, our prediction for  $f_S$  implies substantially larger partial widths of radiative  $J/\psi$  and  $\Upsilon$  decays into scalar glueballs and is therefore important for

experimental glueball searches, in particular for the interpretation of the recent CLEO [8] and forthcoming CLEO-III data on  $\Upsilon \rightarrow \gamma f_0$  and other decay branches.

In the  $0^{-+}$  glueball channel we have found compelling evidence for topological charge screening to provide critical contributions to the IOPE. Besides restoring the axial Ward identity, they are found to resolve the positivity-bound violation and to create a strong signal for both the  $\eta'$  and the pseudoscalar glueball.

After including the screening contributions, all four  $0^{-+}$  Borel-moment sum rules are stable and yield consistent results. (Previous analyses of the  $0^{-+}$  sum rules had discarded the lowest-moment sum rule and therefore missed the chance to implement the first-principle information from the low-energy theorem, as well as a very useful consistency check.) The two-resonance fit is clearly favored over the one-pole approximation, i.e. the IOPE provides clear signals for the  $\eta'$  resonance (with small  $\eta$  admixtures due to mixing) in addition to a considerably heavier  $0^{-+}$  glueball. Our mass prediction  $m_P = 2.2 \pm 1.5$  GeV for the pseudoscalar glueball lies inside the range obtained from quenched and unquenched lattice data. The coupling  $f_P = 0.6 \pm 0.2$  GeV is again enhanced by the nonperturbative Wilson coefficients, but less strongly than in the scalar channel. The consequently larger partial width of radiative quarkonium decays into pseudoscalar glueballs and the enhanced  $\gamma\gamma \rightarrow G_P \pi^0$  cross section at high momentum transfers will be relevant for the experimental identification of the lowest-lying  $0^{-+}$  glueball and help in measuring its properties.

Our nonperturbative IOPE, including the topological short-distance physics, will be useful for calculating other spin-0 glueball properties as well. Quantitative estimates of the mentioned production rates in gluon-rich channels (including  $J/\psi$  and  $\Upsilon$  decays) and characteristic glueball decay properties and signatures, including  $\gamma\gamma$  couplings, OZI suppression and branching fractions incompatible with  $q\bar{q}$  decay, would be particularly interesting.

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